# DIGITAL SIGNAL PROCESSING LABORATORY
## Assignment 5

### 1. Linear Phase Finite Impulse Response (FIR) Frequency Selective Filters

A causal FIR filter which has a real impulse response will have linear phase if and only if the filter has an impulse response of the form

$$h[n] = h[N-1-n] \qquad 0 \le n \le N-1.$$

An FIR filter whose impulse response satisfies this criterion, has a frequency response of the form

$$H(e^{j\omega}) = H(\omega)e^{-j\frac{N-1}{2}\omega}$$

where $H(\omega)$ is a real function and $N$ is the length of the impulse response. If the impulse response is shifted such that $h_S[n] = h[n+(N-1)/2]$, then

$$h_s(n) = \begin{cases} h_s(-n) & -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

where $n \in \mathbb{I}$ for $N$ odd and $n \in \mathbb{I}/2$ (integers divided by 2) for $N$ even. Using the time shifting theorem, the frequency response, $H_s(e^{j\omega})$, of the shifted impulse response, $h_s[n]$, can be written as

$$H_s(e^{j\omega}) = e^{j\frac{N-1}{2}\omega}H(e^{j\omega}) = e^{j\frac{N-1}{2}\omega}H(\omega)e^{-j\frac{N-1}{2}\omega} = H(\omega).$$

Since $H_s(e^{j\omega}) = H(\omega)$, $H_s(e^{j\omega})$ is a real function, and the phase of $H_s(e^{j\omega})$ equals zero. Thus $H(\omega)$ is often called the zero phase frequency response or amplitude response. Given a desired zero phase frequency response, $H_d(e^{j\omega})$, its corresponding impulse response, $h_d[n]$, can be calculated from the inverse discrete time Fourier transform,

$$h_d[n] = \frac{1}{2\pi}\int_{\omega=-\pi}^{\pi} \mathrm{H}_d(\omega)e^{j\omega n}d\omega \text{ ,}$$

where $n \in \mathbb{I}$ for $N$ odd and $n \in \mathbb{I}/2$ (integers divided by 2) for $N$ even. If

$$H_d(\omega) = \begin{cases} 1 & \omega_l \le |\omega| \le \omega_u \\ 0 & \textit{otherwise} \end{cases}$$

then

$$h_d[n] = \frac{\sin[\omega_u n] - \sin[\omega_l n]}{\pi n}$$

and $h_d[n] = h_d[-n]$. In general, the impulse response, $h_d[n]$, is infinitely long. One method used to obtain an FIR filter from $h_d[n]$ is to multiply $h_d[n]$ by a finite duration sequence, $w[n]$, called a window. Therefore the resulting impulse response, $h_r[n]$, is a finite length sequence of the form

$$h_r[n] = h_d[n]w[n] = \begin{cases} h_d[n]w[n] & |n| \le \dfrac{N-1}{2} \\ 0 & |n| > \dfrac{N-1}{2} \end{cases}$$

To guarantee that the frequency response, $H(\omega)$, of the resulting impulse response, $h_r[n]$, is real and has zero phase, the window sequence must have the form

$$w[n] = w[-n]$$

where $n \in \mathbb{I}$ for $N$ odd and $n \in \mathbb{I}/2$ (integers divided by 2) for $N$ even. The following are examples of such windows.

<u>Rectangular Window</u>

$$w[n] = 1 \qquad |n| \le \frac{N-1}{2}$$

<u>Bartlett or Triangular Window</u>

$$w[n] = \begin{cases} \dfrac{2n}{N-1} + 1 & -\dfrac{N-1}{2} \le n \le 0 \\ 1 - \dfrac{2n}{N-1} & 0 \le n \le \dfrac{N-1}{2} \end{cases}$$

<u>Hanning Window</u>

$$w[n] = \frac{1}{2} + \frac{1}{2}\cos\left[\frac{2\pi n}{N-1}\right] \qquad |n| \le \frac{N-1}{2}$$

<u>Hamming Window</u>

$$w[n] = 0.54 + 0.46\cos\left[\frac{2\pi n}{N-1}\right] \qquad |n| \le \frac{N-1}{2}$$

<u>Blackman Window</u>

$$w[n] = 0.42 + 0.5\cos\left[\frac{2\pi n}{N-1}\right] + 0.08\cos\left[\frac{4\pi n}{N-1}\right] \qquad |n| \le \frac{N-1}{2}$$

To obtain a causal FIR filter with impulse response, $h[n]$, from the noncausal FIR filter with impulse response, $h_r[n]$, let

$$h[n] = h_r\left[n - \frac{N-1}{2}\right].$$

The resulting filter will have a frequency response, $H(e^{j\omega})$, where $H_s(e^{j\omega}) = e^{-j\frac{N-1}{2}\omega} H(\omega)$.

## Exercises

1. Modify your MATLAB function, **FIRdesign**, so that it can calculate even and odd length impulse responses.

2. Consider an FIR filter which minimizes the mean square error between the desired zero phase frequency response, $H_d(\omega)$, and the filter's zero phase frequency response, $H(\omega)$, where

$$H_d(\omega) = \begin{cases} 1 & 0.4\pi \le |\omega| \le 0.6\pi \\ 0 & \textit{otherwise} \end{cases}$$

   Determine an impulse response, $h[n]$, of a causal linear phase FIR filter which satisfies this criterion. Let $h[n]$ have the lengths $N$ = 55 and $N$ = 100.

   i) Using **stem**, plot $h[n]$.
   ii) If you used one of the above windows, indicate which window, and plot the magnitude of that window's frequency spectrum.
   iii) Plot the magnitude of the filter's frequency response.
   iv) Give an expression as a function of $\omega$ for filter's phase response.

3. Write MATLAB functions called **bartlett**, **hanning2**, **hamming2** and **blackman** which generate Bartlett, Hanning, Hamming and Blackman windows respectively.

   i) For each of the windows, plot $w[n]$ for $N$ = 55 and $N$ = 100.
   ii) Plot the magnitude of the frequency spectrum of each of the windows when $N$ = 100.

   Compare your **hanning2** and **hamming2** functions to Matlab's built in functions, **hanning** and **hamming**.

4. Using each of the window functions your wrote in Exercise 3, design a series of causal linear phase FIR filters of lengths $N$ = 55 and $N$ = 100 to a approximate the desired frequency response, $H_d(e^{j\omega})$, where

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & 0.4\pi \le |\omega| \le 0.6\pi \\ 0 & \textit{otherwise} \end{cases}$$

   i) Plot the magnitude of the filter's frequency response.
   ii) Give an expression as a function of $\omega$ for filter's phase response.

5. Perform a pole zero plot of one of your filter designs where $N$ = 55.