DIGITAL SIGNAL PROCESSING LABORATORY
Assignment 4

In previous assignments, convolution was used to implement finite impulse response (FIR) linear time invariant (LTI) filters. If the filter had an impulse response of length *N*, then the filter required a maximum *N* multiplies, *N*-1 adds and *2N* memory locations to compute each output . Consequently, convolution is an impractical method for implementing an infinite impulse response (IIR) filter because it would require an infinite number of multiplies, adds and memory locations to compute each output sample. A class of LTI IIR systems can be implemented with recursive linear constant coefficient difference equations (LCCDE).

1. Resonator

a) Determine a LCCDE which has an impulse response of

$$h[n] = r^n \cos(\omega_0 n)u[n]$$

where $\omega_0 = \pi/10$.

b) Using MATLAB's **freqz** function, plot magnitude of the frequency response for this system when $r = 0.99$.

c) Write a MATLAB script to implement your LCCDE. Using this script, plot the first 101 samples of the filter's impulse response for

   i) $r = 0.99$
   ii) $r = 1$
   iii) $r = 1.01$

d) Using your script from part c) of this exercise, calculate the first 251 samples of the zero state response (ZSR) of your difference equation when the input, $x[n]$, is $x[n] = \cos(\omega_0 n)u[n]$ where $\omega_0 = \pi/10$ and

   i) $r = 0.99$
   ii) $r = 1$
   iii) $r = 1.01$

e) Using your script from part c) of this exercise, calculate the first 251 samples of the zero input response (ZIR) of your difference equation when $y[-1] = 4$, $y[-2] = 4$ and

   i) $r = 0.99$
   ii) $r = 1$
   iii) $r = 1.01$

f) Using your script from part c) of this exercise, find the total response of your difference equation when $y[-1] = 4$, $y[-2] = 4$, the input, $x[n]$, is $x[n] = \cos(\omega_0 n)u[n]$ where $\omega_0 = \pi/10$ and

   i) $r = 0.99$
   ii) $r = 1$
   iii) $r = 1.1$

   Add your results from parts d) and e) and compare them to your result in part f). Comment on your comparison.

g) Using MATLAB's **filter** function, repeat part d) of this exercise.

2. IIR Frequency Selective Filters

The following is the system function of a causal lowpass IIR filter

$$H(z) = \frac{0.00183555\left(1+z^{-1}\right)^4}{1-3.05434z^{-1}+3.8289993445z^{-2}-2.29245273626z^{-3}+0.550744355605z^{-4}}.$$

a) Using MATLAB's **freqz** function, plot the frequency response of this system.
b) Using MATLAB's **roots** function, determine the poles of this system. Using either MATLAB or a hand sketch, make a pole zero plot of $H(z)$. Assuming that the system is causal, what can you say about the filter's stability and why?
c) Generate and plot the first 101 samples of the following sequences.

   i) $a[n] = \cos[(\pi/8)\,n]u[n]$
   ii) $b[n] = \cos[(\pi/3)\,n]u[n]$
   iii) $c[n] = \cos[(\pi/8)\,n]u[n] + \cos[(\pi/3)\,n]u[n]$

d) Using MATLAB's **filter** function, filter signals $a[n]$, $b[n]$ and $c[n]$. Plot your results. Explain your results.

3. Allpass System

Consider the following allpass system function

$$H_{ap}(z) = \frac{z^{-1}-a}{1-az^{-1}} \quad \frac{\left(bb^*\right)-\left(b+b^*\right)z^{-1}+z^{-2}}{1-\left(b+b^*\right)z^{-1}+bb^*z^{-2}}$$

where $a = 0.9$, $b = 0.9\exp(j\pi/3)$ and the symbol * denotes complex conjugate. (Note that you will need to use .' and not ' to transpose vectors with complex elements.)

a) Using MATLAB's **freqz** function, plot the frequency response (both magnitude and phase). Using MATLAB's **grpdelay** function, plot the group delay of this system.

b) Generate and plot the first 101 samples of the sequence, $d[n]$, where

$$d[n] = 1/2 + \frac{1}{\pi}\sum_{k=1}^{7}\frac{1}{k}\sin\left(k\omega_0 n\right)$$

   and $\omega_0 = \pi/10$. Also plot the magnitude of the frequency spectrum of $d[n]$.

c) Using MATLAB's **filter** function, filter signal $d[n]$. Plot the output sequence and the magnitude of its frequency spectrum. Comment on your results.