# DIGITAL SIGNAL PROCESSING LABORATORY
## Assignment 3

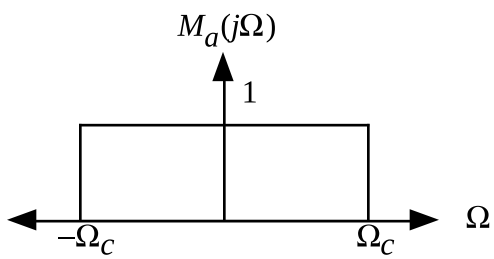
### 1. Sampling

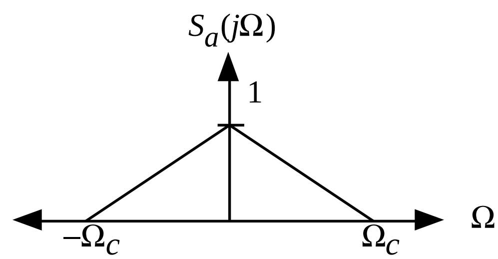
Digital signals can be obtained by sampling analog signals. Ideal sampling is performed by a device called a continuous to discrete time (C/D) converter. Sampling an electrical signal such as a current or voltage is often accomplished by a device called an analog to digital (A/D) converter. Ideal sampling of an analog signal is the act of evaluating an analog signal at a discrete set of times called sample times. Sampling an analog signal $x(t)$ can be mathematically represented by

$$x[n] = x_a(t)\big|_{t=nT} = x_a(nT) \qquad -\infty < n < \infty.$$

where $x[n]$ is the discrete time signal acquired by sampling the analog signal, $x_a(t)$, and $T$ is the *sampling period* defined as the period of time between samples. $T$ is usually constant. The reciprocal of the sampling period is defined as the *sampling frequency* or the *sampling rate*, and is denoted as $f_s$. If $T$ is expressed in seconds per sample, then $f_s$ is expresses in samples per second or Hz.

a) Consider the following bandlimited analog signals which have the following frequency spectrums




$$M_a(j\Omega) = \begin{cases} 1 & |\Omega| \le \Omega_c \\ 0 & |\Omega| > \Omega_c \end{cases}$$

$$S_a(j\Omega) = \begin{cases} -\dfrac{1}{\Omega_c}\Omega + 1 & 0 \le \Omega \le \Omega_c \\ \dfrac{1}{\Omega_c}\Omega + 1 & -\Omega_c \le \Omega \le 0 \\ 0 & |\Omega| > \Omega_c \end{cases}$$

where $\Omega_c = 300\pi$ rad/sec $\Rightarrow f_c = 150$ Hz. Using the inverse continuous time Fourier transform,

$$x_a(t) = \frac{1}{2\pi}\int_{\Omega=-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t}\, d\Omega,$$

determine $m_a(t)$ and show that

$$s_a(t) = \frac{1 - \cos(\Omega_c t)}{\pi \Omega_c t^2}.$$

b) Using MATLAB, sample $m_a(t)$ and $s_a(t)$ for $-1 \le t \le 1$ at the following sampling rates

i) $f_s$ = Nyquist rate
ii) $f_s$ = 900 Hz
iii) $f_s$ = 200 Hz

Using the built in Matlab function **freqz**(m, 1, 512) and **freqz**(s, 1, 512) calculate and plot the frequency spectrum of $m(n)$ and $s(n)$ acquired in parts i), ii) and iii). Comment on your plots.

## 2. Sampling Rate Conversion

Systems that process signals at multiple sampling rates are called *multirate systems*. The process of digitally converting the sampling rate of a signal to a different sampling rate is called *sampling rate conversion*. Sampling rate conversion is performed using systems called *decimators* and *interpolators*.

Figure 1 shows a system called a decimator which reduces the sampling rate of a signal by an integer factor of $M$. As shown in Figure 1, a decimator consists of a cascade of a lowpass filter and a compressor (or downsampler). If the signal $x_{lp}[n]$ is the input to the compressor, then the output, $x_d[n]$, will be a sampled version of $x_{lp}[n]$ such that

$$x_d[n] = x_{lp}[nM] \qquad -\infty < n < \infty..$$

A decimator reduces the number of samples in the sequence by a factor of $M$. The usage of the term *decimator* is not consistent throughout literature. The term *decimator* is often used to refer to the *compressor*. In this assignment, the term, *decimator,* will be used to refer to the combination of filtering and downsampling.

a) Consider the signals sampled in exercise 1, section b) part ii). Design a decimator that will reduce the sampling rate to the Nyquist rate. Using MATLAB, implement this system, and plot the frequency spectra of the decimated signals. (Note: Because $m[n]$ and $s[n]$ are bandlimited, a lowpass filter is not necessary for this problem. However if you choose to add a filter to this problem, use your **FIRdesign** function to build a FIR filter of length $N$ where $N \ge 51$. Then use any of your convolution functions to filter the signal.)

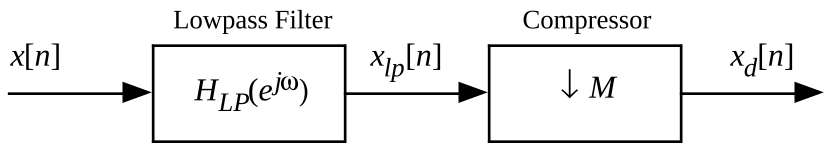



Figure 1. A decimator which reduces the sampling rate of the signal $x[n]$ by an integer factor of $M$.

Figure 2 shows a system called an interpolator that digitally increases the sampling rate (interpolating) of a signal by an integer factor of $L$. This system increases the sampling rate by interpolating $L$-1 samples between each pair of samples in the signal $x[n]$. As shown in Figure 2, an interpolator consists of a cascade of an expander (or upsampler) and a lowpass filter. If the signal $x[n]$ is the input to the expander, then the output, $x_e[n]$, will be $x[n]$ with $L$-1 zeroes inserted between each pair of samples of $x[n]$ such that

$$x_e[n] = \begin{cases} x[n/L] & n = 0, \pm L, \pm 2L, \pm 3L \cdots \\ 0 & \text{otherwise} \end{cases}$$

An interpolator increases the number of samples in the sequence by a factor of $L$. The usage of the term *interpolator* is not consistent throughout literature. The term *interpolator* is often used to refer to the *expander*. In this assignment, the term, *interpolator*, will be used to refer to the combination of upsampling and filtering.

b) Consider the signals sampled in exercise 1, section b) and part i). Design an interpolator that will increase the sampling rate to 900 Hz. Using Matlab, implement this system and plot the frequency spectra of the interpolated signals. (Use your **FIRdesign** function to build a FIR filter of length $N$ where $N \geq 51$. Then use any of your convolution functions to filter the signal.)

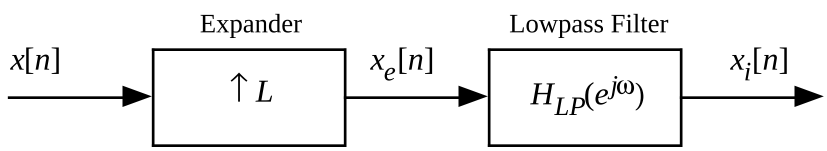



Figure 2. An interpolator which increases the sampling rate of the signal $x[n]$ by an integer factor of $L$.

The frequency division multiplexing (FDM) digital communication system shown in Figure 3 can transmit several digital signals simultaneously over a single channel by transmitting each signal over a finite bandwidth of frequencies. Each message signal $m_1[n]$, $m_2[n]$, ..., $m_k[n]$ is upsampled and then filtered to fit in its assigned frequency band. The interpolated signals are added together and then transmitted across a single channel. At the receiver, the incoming signal is filtered so that unwanted signals are suppressed and the desired signal is passed. The filtered channel signal is then downsampled to recover the desired message signal.

c) Consider the signals sampled in exercise 1, section b) and part i). Design and implement an interpolator that will bandlimit $m(n)$ to the frequencies $|\omega| < \pi/3$ and bandlimit $s(n)$ to the frequencies $\pi/3 < |\omega| < \pi$. Add the two interpolated signals together and plot the resulting frequency spectrum. Design and implement a decimator that will recover $m(n)$ and $s(n)$ at the original sampling rates. Plot the resulting frequency spectrums.

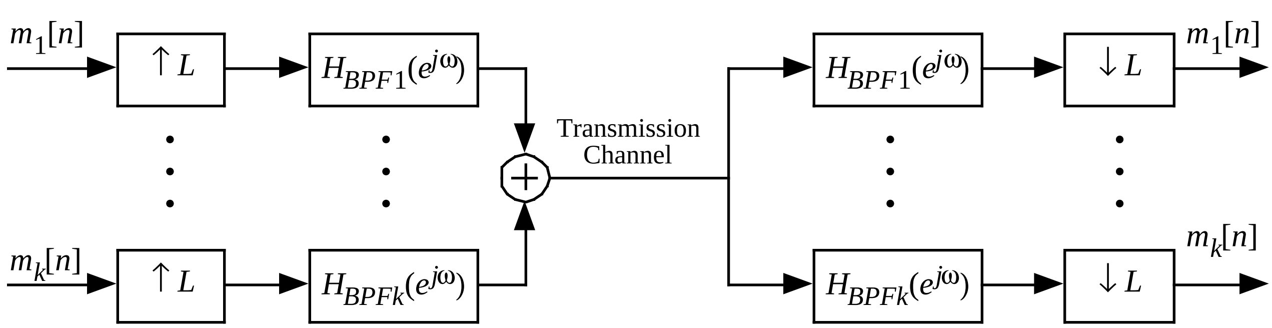



Figure 3. A frequency division multiplexing (FDM) digital communication system implemented with interpolators and decimators.

## 3. Changing the Sampling Rate by a Noninteger Factor

Sample the signal $x_a(t) = \cos(\Omega_m t)$ at 400 Hz for $0 \le t \le 0.25$ where $\Omega_m = 40\pi$. Using **stem**, plot $x[n]$. Design and implement a multirate system that will digitally change the sampling rate to 300 Hz. Using **stem**, plot the resulting signal.