DIGITAL SIGNAL PROCESSING LABORATORY
Assignment 2

1. Convolution

A system's impulse response, often denoted $h[n]$, is defined as the system's output sequence when the system's input is the impulse sequence, $\delta[n]$. If a system can be classified as a linear time invariant (LTI) system, then the system is completely characterized by its impulse response. The output, $y[n]$, of any LTI system can be determined by the convolution of the system's impulse response, $h[n]$, and its input sequence, $x[n]$. This operation is commonly written as

$$y[n] = h[n] * x[n]$$

and is mathematically defined as

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

a) Using the **shift**, **reverse** and **compsig** functions that you wrote in the first assignment, write a MATLAB function called **convolve** which has the form,

[y,n] = convolve(h,nh,x,nx),

where **h** is the system's impulse response, **nh** is the time index for **h**, **x** is the input sequence, **nx** is the time index for **x**, **y** is the convolution of **h** and **x** and **n** is the time index for **y**. The function **convolve** should be able to handle causal and noncausal impulse responses and input signals. To determine starting and ending time for $y[n]$.

To test your function, consider the signal

$$x[n] = u[n] - u[n-5] .$$

Notice that $x[n]$ is a finite length sequence of length 5 and can be generated using

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x = [1 1 1 1 1]' ;  nx = 0:4;  nx = nx'
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Use your function, **convolve**, to perform the following convolutions:

i) $y_1[n] = x[n] * x[n]$
ii) $y_2[n] = x[n-1] * x[n-1]$
iii) $y_3[n] = x[n+1] * x[n+1]$
iv) $y_4[n] = x[n+1] * x[n-1]$

Plot your results. To verify that your program works correctly, perform the same convolutions on scratch paper using the graphical convolution method. Your scratch paper does not need to put into your report.

b) Write a MATLAB function called **convolution** which has the form,

y = convolution(h),

where **h** is the system's impulse response and **y** is the convolution of **h** and an input sequence entered from the keyboard. The purpose of this function is to filter an input sequence in real time. The function should accept an input sample from the keyboard, display the corresponding output value and prompt the user for another input. If the impulse response, **h**, has a length *N*, the function, **convolution**, should require no more than *N* multiplies and *N*-1 additions per output sample. This function should be performing convolution in real time. Thus, the impulse response, the input sequence and the output sequence are assumed to be causal and will have an implied common time index starting at time zero.

Repeat the exercises i) and ii) in part a) using your function, **convolution**. Plot your results.

c) The function **conv**, which is provided in the MATLAB toolbox, also performs convolution. Repeat the exercises i) and ii) in part a) using the **conv** function. Plot your results.

2. Finite Impulse Response (FIR) Frequency Selective Filters

An ideal frequency selective filter is a system which passes certain frequency components of a signal and rejects others. Given the desired frequency response, $H_d(e^{j\omega})$, of a frequency selective filter, its corresponding impulse response, $h_d[n]$, can be calculated from the inverse discrete time Fourier transform,

$$h_d[n] = \frac{1}{2\pi}\int_{\omega=-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega \ .$$

Ideal frequency selective filters generally have abrupt changes and discontinuities in their desired frequency responses. As a result, the impulse responses of ideal frequency selective filters are usually noncausal and infinitely long. One method used to obtain a FIR filter from $h_d[n]$ is to multiply $h_d[n]$ by a finite duration sequence, $w[n]$, called a window. Therefore, the resulting impulse response, $h_r[n]$, is a finite length sequence of the form

$$h_r[n] = h_d[n]w[n] = \begin{cases} h_d[n]w[n] & |n| \le \frac{N-1}{2} \\ 0 & |n| > \frac{N-1}{2} \end{cases}.$$

If $H_d(e^{j\omega})$ is a real function, the phase of $H_d(e^{j\omega})$ equals zero, and $h_d[n] = h_d[-n]$. To preserve the zero phase characteristics, a window is designed to have the form, $w[n] = w[-n]$, so that the resulting impulse response, $h_r[n]$, has the form $h_r[n] = h_r[-n]$. One such window is called the rectangular or box car window and is defined as

$$w[n] = \begin{cases} 1 & |n| \le \frac{N-1}{2} \\ 0 & |n| > \frac{N-1}{2} \end{cases}$$

The rectangular window emulates truncation, preserving the Fourier coefficients. Thus, the resulting impulse response, $h_r[n]$, is the optimal impulse response in the sense that it minimizes the mean square error between $H_d(e^{j\omega})$ and $H_r(e^{j\omega})$. To obtain a causal FIR filter with impulse response, $h[n]$, from the noncausal FIR filter with impulse response, $h_r[n]$, let

$$h[n] = h_r\left[n - \frac{N-1}{2}\right].$$

The resulting filter will have a frequency response, $H(e^{j\omega})$, where $H(e^{j\omega}) = e^{-j\omega\frac{(N-1)}{2}} H_r(e^{j\omega})$

a) Consider the following ideal frequency selective filter

$$H_d(e^{j\omega}) = \begin{cases} 1 & \omega_l \le |\omega| \le \omega_u \\ 0 & \omega_u < |\omega| \le \pi \;\cup\; |\omega| < \omega_l \end{cases}$$

Write a MATLAB function called **FIRdesign** which calculates the impulse response, $h[n]$, of the causal FIR filter that minimizes the mean square error between $H_d(e^{j\omega})$ and $H(e^{j\omega})$. The function should have the form

h = **FIRdesign(wl,wu,N)**,

where **h** is the causal FIR filter's impulse response, **wl** = ω_l, **wu** = ω_u, and **N** is the length of the rectangular window sequence, $w[n]$.

b) Consider a FIR filter which minimizes the mean square error between the desired frequency response, $H_d(e^{j\omega})$, and the filter's frequency response, $H(e^{j\omega})$, where

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\omega| \le \pi \end{cases}$$

Determine the filter's impulse response, $h[n]$, when $h[n]$ has the following lengths:

i) $N = 15$
ii) $N = 33$
iii) $N = 99$

Use the **[H,w] = freqz(h,1,n)** function provided by MATLAB to determine the frequency response of each of the filters. Plot the magnitude and phase of each filter.

c) Generate and plot the first 101 samples of the following sequences.

i) $a[n] = \cos[(\pi/8)\,n]u[n]$
ii) $b[n] = \cos[(\pi/3)\,n]u[n]$
iii) $c[n] = \cos[(\pi/8)\,n]u[n] + \cos[(\pi/3)\,n]u[n]$

Using any of the filters that you designed in part b) of this exercise and any of the convolution functions you used in Exercise 1, filter signals $a[n]$, $b[n]$ and $c[n]$. Plot your results. Explain your results.